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Towards the issue of the methods of calculating fractal dimension of the disperse pulverescent-clayey soil structure

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Abstract. Our study on the structure of pulverescent-clayey loess deposits was conducted at different levels of organization with the purpose of clarifying the characteristics of their deformational behaviour. In our description of the structure and microstructure, the theory of fractals is used in the study of macroporosity, inter-aggregate porosity and interparticle porosity of loess

loams and sand loams. The calculations of indicators of fractal dimension structure can be made using analysis of electronic images of microstructure, granulometry and dispersity of soils, experimental calculations of penetrability and porosity. Comparison of the results of quantitative fractal dimension indicators obtained using different methods is of interest at the stage of choosing a more efficient and meaningful method. The article analyses the results of calculating the values of the fractal dimension of particle distribution function, which were obtained using different methods. We conducted analysis of discretion of differently formed stratigraphic-genetic horizons. The images were analyzed in the "JmagerL" programme, the calculation of the Hurst exponent and fractal dimension of the particle distribution function was made using "Fraktan" software and EXCELL electronic tables. The study analyzed images on different scales. The results of the calculations of the particle sizes were considered as a series of fractal dimension series sufficient for calculation, Hurst exponent, correlational dimension and entropy of length. The results of granulometric examination (pipette method, three methods for preparing the samples) were analyzed using classified data on the fractions. The following fractions were considered: the fractions with particles less than 0.001 mm, 0.002 mm, 0.005 mm, 0.01 mm, 0.05 mm, 0.1 mm, 0.25 mm, 0.5 mm, 1 mm, 2 mm, 5 mm, 10 mm. The values of fractal dimension of particle distribution at the level of inter-aggregate and interparticle porosity were different. When the disperse method for preparing the sample was used, the fractal dimension of particle (fraction) distribution obtains higher values compared to the aggregate and semi-disperse methods. We proved the relation between the values of fractal dimension and the genesis, discretion and method for preparing the soil samples. The dimension values of loess sediments were lower compared to paleosol regardless of the method for the preparation. Comparability of the assessments of fractal dimension of distribution of the solid phase of different topological dimension (flat sections and volumetric particles) was observed during aggregate and semi-disperse methods of preparation.

Keywords: loess, porosity, fractal dimension, particle distribution function.

До питання про методи оцінки фрактальної розмірності структури дисперсного пилувато-глинистого ґрунту.

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Резюме. Вивчення структури пилувато-глинистих лесовидних відкладів виконується на різних рівнях організації з метою уточнення особливостей їх деформаційної поведінки. Застосування теорії фракталів до опису структури і мікроструктури виконується при вивченні макропористості, міжагрегатної і міжчасткової пористості лесовидних суглинків і супісків. Кількісні оцінки значень фрактальної розмірності структури можна отримати при аналізі електронних знімків мікроструктури, гранулометрії і дисперсності ґрунтів, експериментальних оцінок проникності і пористості. Зіставлення результатів кількісних оцінок фрактальної розмірності, отриманих різними методами, становить інтерес на етапі вибору більш результативного і обґрунтованого методу. У даній роботі проаналізовані результати кількісної оцінки значень фрактальної розмірності функції розподілу часток, які отримані різними методами. Виконано аналіз дискретності різних за умовами походження стратиграфо-генетичних горизонтів. Обробка знімків виконувалася за допомогою програми "JmagerL", розрахунок показника Херсту і фрактальної розмірності функції розподілу часток виконувався за допомогою ПО "Fraktan" і електронних таблиць EXCELL. Досліджено знімки в різних масштабах. Поодинокій запис результатів підрахунку розмірів частинок розглядався як ряд достатньою для розрахунку фрактальної розмірності ряду, показника Херсту, кореляційної розмірності і

ентропії довжини. Обробка результатів гранулометричного аналізу (піпеточний метод, три способи підготовки зразків) виконувалася по згрупованим даними - фракціям. враховувалося змiст частинок розмірами менше 0,001мм, 0,002мм, 0,005мм, 0,01мм, 0,05мм, 0,1мм, 0,25 мм, 0,5 мм, 1 мм, 2 мм, 5 мм, 10 мм. Значення фрактальної розмірності розподілу часток на рівні міжагрегатної і міжчасткової пористості відрізняються. Фрактальна розмірність частинок (фракцій) при дисперсному способі підготовці зразка приймає більш високі значення, ніж за агрегатної або полідисперсної підготовкою. Підтверджено залежність значень фрактальної розмірності від генезису, дискретності і способу підготовки зразка ґрунту. Значення розмірності приймають менші значення в лесових відкладах, ніж в палеоґрунтах, незалежно від способу підготовки. Порівнянність оцінок фрактальної розмірності розподілу твердої фази різної топологічної розмірності (плоских перетинів і об'ємних часток) спостерігається при агрегатному і полідисперсному способах підготовки.

Ключові слова: лес, пористість, фрактальна розмірність, функція розподілу часток.

The problem statement and state of study. The usage of the theory of fractals in studying natural objects, phenomena and processes has rapidly increased in recent years Puetz, S.J., Borchardt G. (2015); Cheng Wen-Chieh et al, (2017); X. Du · et al, (2017).

The study of pulverescent-clayey soils as a natural multifractal system is a matter of great of interest (Zhan Ce Wang et al, 2009). The description of soil structure from the perspective of the theory of fractals, search for relation between the genesis and peculiarities of the structure, attempts of defining patterns in deformation and decrease in the strength of soils as natural fractals is studied by scientists of various countries (Russell AR, 2014; Muñoz-Castelblanco JA and all, 2012; Li P. et al, 2016).

The classical methods of predicting the deformation of soil environments are based on using the model of continuous medium, and less often on the discrete medium model; the processes are considered reversible (Jiang M. and all, 2017). A principal difference of the non-classic method is defining patterns of deformation in soil as a natural fractal, a dynamic evolving system (Ikeda, Kiyohiro et al, 2008), capable of chaotic behaviour.

Methods. We studied the peculiarities of the structure of loess sediments in Prychornomorsko-Dofinovski, Vitachevski, Udaiski, Dniprovski deposits from the perspective of the theory of fractals. The samples were collected from natural outcrops around the city of Dnipro: in the slopes of the Yevpatoriiska, Tonnalna, and Sazhenka ravines and from the Rybalsky Quarry.

The basin of the Yevpatoriiska ravine includes Aeolian-diluvial loam sands of the Tiasmynsky horizon (*vd P_{11ts}*), Dniprovski Aeolian-diluvial loess loam sands (*vd P_{11dn}*) and Zavadovskieluvial loams (*e P_{11zv}*). In the basin of the Tonnalna ravine, we collected samples of the Prychornomorsko-Dofinovsky Aeolian horizon (*vd P_{III pc + df}*), Buhskieluvial-diluvial loam sands (*vd P_{IIbfg}*), Dniprovski Aeolian-diluvial loam sands (*vd P_{11dn}*) (Fig. 4.1 a). The samples of Buhski Aeolian-diluvial loam sands (*vd P_{IIbfg}*), eluvial loam sands of the Vitachevsky horizon (*e P_{IIIvt}*), Aeolian-

diluvial loam sands of the Dniprovsky horizon (*vd P_{11dn}*) were also collected in the basin of the Sazhenka ravine.

We calculated the full complex of indicators of physical properties, focusing mainly on the characteristics of dispersity. The calculation of fractal dimension of the surfaces, images, **ramifield** systems is performed in materials science, medicine, geology using the results of analyzing a series of data of sufficient length. The digital model of an electronic image of the soil structure can be converted into a series of values of area, diameter, perimeter and contour (of particles and pores), using the corresponding programme, in particular, "JmageL".

The developed sequence is a random series of the contours' occurrence on the surface, i.e. a characteristic of microstructure. The analysis of the images made on different scales allows analysis of different elements of the soil structure – aggregates, particles and pores. The sufficient length of the series allows the calculation of the fractal dimension of a row, Hurst exponent, correlational dimension and entropy ["Fraktan" software].

Due to anisotropy and complex form of particles, aggregates and pores, the calculations of fractal dimension of random section do not allow accurate characterization of the soil structure, but allow making selective estimations of distribution of the number of particles of particular size.

Russell A R (2011) has suggested a method of calculating fractal dimension of function of particles distribution by using the results of standard granulometric analysis. This case does not include the calculation of plainly visible particles. Loess pulverescent-clayey soils are characterized by predisposition of the microaggregate to the break-up under the change of external conditions, which is the manifestation of their degradation during the process of anthropization. This method (Riashenko T H, 2010) allows one to obtain more accurate calculations of content of free particles and particles of different fractions assembled in aggregates than when using standard granulometric analysis.

The main results of the study. We performed the calculations of fractal dimension of function of par-

ticle distribution during the analysis of plane images and on the basis of results of analysis of samples' microstructure with calculation of composition of aggregates and particles of loess from the Dniprovsky horizon. Electronic images were made using an electronic microscope at 80, 200 and 500 x

magnification (Fig. 1). The analysis of images was made using the programme "JimageL". The calculations of the Hurst exponent, correlational dimension, fractal dimension of function of distribution of particles and pores were performed using "Fractan" programme.

Fig. 1. The structure of a sample of loess loam *vdP_{II}dn* of Dniprovsky horizon A) 80x magnification B) 200x magnification.

During the preparation of the initial image for the analysis of particles, we used operations of calibration, scaling, conversion of the image. We performed the calculations of radius and diameters of particles, number of particles included at a certain interval. The values of intervals were set in correspondence with current classification of the dispersed rocks; the content of particles with sizes less than 0.001 mm, 0.002 mm, 0.005 mm, 0.01 mm, 0.05 mm, 0.1 mm, 0.25 mm, 0.5 mm, 1 mm, 2 mm, 5 mm, 10 mm were considered. If the particles of these sizes were not visible in the image, the fraction was omitted.

The second stage was calculation of the fractal dimension of the random ordered sequence of the image contours (form 1) and the random un-

ordered sequence (form. 2). In the first case, the data was grouped according to the fractions; the second case used no grouping methods. Charts of dependencies between the total number of particles in the fraction or the number of particles and the diameter (fig. 2, 3) were developed. The fractal dimension of the grouped data D_s was calculated using the formula:

$$D_s = 2 - k \quad (\text{form. 1})$$

Where D_s – fractal dimension of the distribution of the particles' contours;
2 – topological dimension of the image;
K – the slope of the line $\ln N_R = f(\ln D_R)$.

Fig. 2. The chart of relation between the total number of particles N_R and the diameter D_R in the form of $\ln N_R = f(\ln D_R)$ (Udaiski, *vdP_{II}ud*).

Notes to Fig. 2:

1. N_R – the total number of particles; D_R – the largest diameter
2. R^2 – determination coefficient
3. x– experimental values; — - linear trend.

The fractal dimension of random sequence was calculated using the Hurst exponent H (form. 2).

$$D = 2 - H \quad (\text{form. 2}).$$

The values of fractal dimension D and D_s are significantly different, which indicates the difference in

the calculations of fractal dimension of functions of fraction and certain particle distribution (Table 1). The values of dimension change in relation to the increase, i.e. the type of porosity (macroporosity or inter-particle porosity).

Selected results of calculations of values of fractal dimension of the structure of the Dniprovsky horizon
Table 1.

Magnification	The fractal dimension of the function of particle (fraction) size distribution		The fractal dimension of the particle contour sequence	
	k	D_s	D	H
80	0.045	1.955	1.119	0.881
200	0.034	1.966	1.143	0.857
800	0.037	1.963	1.145	0.855

Notes to Table 1.

K – slope coefficient of the straight line; D_s – fractal dimension of the sequence of grouped data; D – fractal dimension of contour sequence; H – the Hurst exponent

The calculations of the dimension of the function of particle distribution using the results of granulometric analysis with three methods of sample preparation indicate the presence of a dependency between the dispersity (aggregativity) and the dimension.

The sample preparation method also affects the dispersity of soil (Fig.3). The disperse method of preparation achieves the highest decomposition of the aggregates and higher number of fine fractions compared to the samples prepared using aggregate or semi-disperse method.

Fig. 3. Integral curves of granulometric composition of Dofinovskyy $edPIIIdf$, Udaisky $vdPIIId$ and Dniprovsky $vdPIIdn$ horizons.

Notes to Fig. 3, 4.

D – disperse method of sample preparation; A – aggregate method of sample preparation; II – semi-disperse method of sample preparation.

The fractal dimension of fraction distribution function was calculated using the method described above (see form. 1, Fig. 2), the value of topological dimension was considered as equal to 3. The analysis of the results showed that using the disperse method of sample preparations, the values

of the fractal dimension of the particles (fractions) are higher compared to the aggregate and semi-aggregate methods (Fig. 4).

This indicates the natural relation between the dispersity and the fractal dimension of the particle distribution function of loess soil.

Fig. 4. The fractal dimension of Dofinovskiy $edP_{III}df$, Udaiskiy $vdP_{III}ud$ and Dniprovskiy $vdP_{II}dn$ horizons

Also we calculated the fractal dimension of the distribution of particles in the samples with altered condition: compressed and filtrated (Table 2). The fractal dimension of the distribution of particle sizes is related not only to the dispersity, but also to the hydrophility. In the horizons of fossil soils (water-saturated condition) the values of dimension were higher, and in the subaerial loess sediments were lower compared to the dimension typical for natural conditions. A sharp rise in the content of free clayey particles as a result of compression in

the water-saturated condition is typical for the horizons of fossil soils. The loess sediments were observed to have change in the content of pulverescent and larger particles and aggregates.

As a result, the value of the fractal dimension were lower after compression and moisturizing of the soil. Usage of magnetic stirrer in the preparation of the sample for the granulometric analysis leads to increase in the value of dimension compared to the values after using mechanical stirring

Table 2. The fractal dimension of the soil particles distribution function (change in the condition).

Horizon	S	Experimental conditions			
		V	Z	M	F
Prychornomorsko-Dofinovskiy ($vd, eP_{III}pc+df$)	2.7	2.67	2.63	2.41	–
Udaisky horizon ($vd P_{III}ud$)	2.25	–	–	2.61	–
Tiasmysky horizon ($vdP_{II}ts$)	2.45	–	–	–	2.37
Dniprovsky horizon ($vd P_{II} dn$)	2.57	–	–	2.65	–
	2.59	2.73	–	2.67	–
	2.76	–	–	–	2.45
Kaidaksky horizon ($e P_{II}kd$),	2.78	2.39	2.32	–	–
Zavadovskyhorizon ($eP_{II}zv$)	2.62	–	–	–	2.5

Notes to table 2:

S – standard preparation of the sample for granulometric analysis;
V – the condition after the compression in water-saturated condition;
Z – the sample tested for the subsidence using the method of a single curve;
M – aggregate preparation (magnetic stirrer);
F – the sample affected by the filtration.

Results and discussion. The method of sample preparation for granulometric analysis affects the measurement of the content of fine fractions, fractal dimension of particles distribution function. Disregarding the methods of preparation, the values of dimension are lower in loess sediments compared to the paleosols.

Comparability of the values of fractal dimension of distribution of solid phase of different topological dimension (flat sections and volumetric particles) was observed during the aggregate and semi-disperse methods of preparation.

The external effect on the loess soil causes a change in the microaggregate structure, which was observed in the changes of fractal dimension of the soil particle distribution function.

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